

The Linguistic Interpretation of Quantum Mechanics

Shiro Ishikawa

*Department of Mathematics, Faculty of Science and Technology, Keio University,
3-14-1 Hiyoshi, Kohoku-ku, Yokohama, 223-8522 Japan (E-Mail: ishikawa@math.keio.ac.jp)*

About twenty years ago, we proposed the mathematical formulation of Heisenberg's uncertainty principle, and further, we concluded that Heisenberg's uncertainty principle and EPR-paradox are not contradictory. This is true, however we now think that we should have argued about it under a certain firm interpretation of quantum mechanics. Recently we proposed the linguistic quantum interpretation (called quantum and classical measurement theory), which was characterized as a kind of metaphysical and linguistic turn of the Copenhagen interpretation. This turn from physics to language does not only extend quantum theory to classical systems but also yield the quantum mechanical world view (i.e., the philosophy of quantum mechanics, in other words, quantum philosophy). In fact, we can consider that traditional philosophies have progressed toward quantum philosophy. In this paper, we first review the linguistic quantum interpretation, and further, clarify the relation between EPR-paradox and Heisenberg's uncertainty principle. That is, the linguistic interpretation says that EPR-paradox is closely related to the fact that syllogism does not generally hold in quantum physics. This fact should be compared to the non-locality of Bell's inequality.

1. Introduction

About twenty years ago (in 1991), we proposed and proved the mathematical formulation of Heisenberg's uncertainty principle (*cf.* Theorem 2 and Corollary 1 in ref. [1]). As mentioned in Section 4.6 (i.e., inequalities (8) and (9)) later, note that Heisenberg's uncertainty principle should not be

confused with Robertson's uncertainty principle [2]. And further, in the remark 3 of [1], we discussed the relation between Heisenberg's uncertainty principle and EPR-paradox [3], and concluded that the two are not contradictory. This is true, but now we consider that the argument in the remark 3 of [1] is somewhat shallow. In order to clarify the relation between Heisenberg's uncertainty principle and

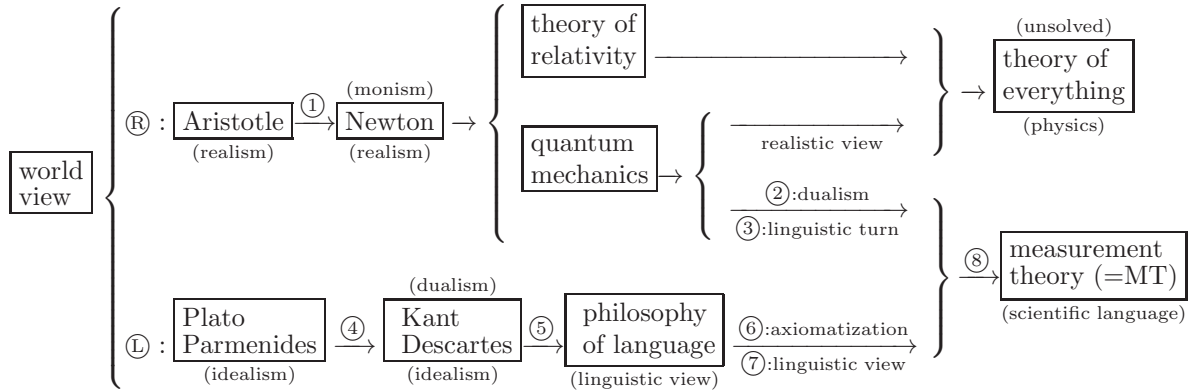


Figure 1. Traditional philosophies from the quantum philosophical point of view (*cf.* Section 5)

EPR-paradox, we must start from the proposal of the interpretation of quantum mechanics. That is because even the Copenhagen interpretation is not determined uniquely. This is our motivation to propose the linguistic interpretation of quantum mechanics [4, 5].

In this paper, adding new examples, we first review the linguistic interpretation [4, 5], and explain **Figure 1**. And lastly, we clarify the relation between EPR-paradox and Heisenberg's uncertainty principle in the linguistic interpretation of quantum mechanics.

2. Measurement Theory (Axioms)

2.1. Overview: Measurement theory

In this section, we shall explain the overview of measurement theory (or in short, MT).

Measurement theory (refs. [4–11]) is, by an analogy of quantum mechanics (or, as a linguistic turn of quantum mechanics), constructed as the scientific theory formulated in a certain C^* -algebra \mathcal{A} (i.e., a norm closed subalgebra in the operator algebra $B(H)$ composed of all bounded operators on a Hilbert space H , cf. [12, 13]). MT is composed of two theories (i.e., pure measurement theory (or, in short, PMT) and statistical measurement theory (or, in short, SMT). That is, we see:

(A) MT (measurement theory)

$$\begin{aligned}
 &= [\text{measurement}] + [\text{causality}] \\
 &\quad \quad \quad \text{(Axiom 1)} \quad \quad \quad \text{(Axiom 2)} \\
 &= \begin{cases} (A_1) : [\text{PMT}] \\ \quad = [(\text{pure}) \text{ measurement}] + [\text{causality}] \\ \quad \quad \quad \text{(Axiom}^P \text{ 1)} \quad \quad \quad \text{(Axiom 2)} \\ (A_2) : [\text{SMT}] \\ \quad = [(\text{statistical}) \text{ measurement}] + [\text{causality}] \\ \quad \quad \quad \text{(Axiom}^S \text{ 1)} \quad \quad \quad \text{(Axiom 2)} \end{cases}
 \end{aligned}$$

where Axiom 2 is common in PMT and SMT. For completeness, note that measurement theory (A) (i.e., (A₁) and (A₂)) is a kind of language based on “the quantum mechanical world view”. It may be understandable to consider that

(B₁) PMT and SMT is related to Fisher's statistics and Bayesian statistics respectively.

This is discussed in [6]. Thus, if we believe in **Figure 1**, we can answer to the following problem (cf. [6]):

(B₂) What is statistics? Or, where is statistics in science?

2.2. Mathematical preparations

In this paper, we mainly devote ourselves to the C^* -algebraic formulation, and not the W^* -algebraic formulation (cf. the appendix in [4]). Thus, Axiom^P 1 is often denoted by Axiom 1.

When $\mathcal{A} = B_c(H)$, the C^* -algebra composed of all compact operators on a Hilbert space H , the (A) is called quantum measurement theory (or, quantum system theory), which can be regarded as the linguistic aspect of quantum mechanics. Also, when \mathcal{A} is commutative (that is, when \mathcal{A} is characterized by $C_0(\Omega)$, the C^* -algebra composed of all continuous complex-valued functions vanishing at infinity on a locally compact Hausdorff space Ω (cf. [12])), the (A) is called classical measurement theory. Thus, we have the following classification:

$$(C) \quad \text{MT} \begin{cases} \text{quantum MT} & (\text{when } \mathcal{A} = B_c(H)) \\ \text{classical MT} & (\text{when } \mathcal{A} = C_0(\Omega)) \end{cases}$$

In this paper, we mainly devote ourselves to classical MT (i.e., classical PMT and classical SMT).

Now we shall explain the measurement theory (A). Let $\mathcal{A}(\subseteq B(H))$ be a C^* -algebra, and let \mathcal{A}^* be the dual Banach space of \mathcal{A} . That is, $\mathcal{A}^* = \{\rho \mid \rho \text{ is a continuous linear functional on } \mathcal{A}\}$, and the norm $\|\rho\|_{\mathcal{A}^*}$ is defined by $\sup\{|\rho(F)| : F \in \mathcal{A} \text{ such that } \|F\|_{\mathcal{A}} (= \|F\|_{B(H)}) \leq 1\}$. The bi-linear functional $\rho(F)$ is also denoted by $\mathcal{A}^* \langle \rho, F \rangle_{\mathcal{A}}$, or in short $\langle \rho, F \rangle$. Define the *mixed state* $\rho (\in \mathcal{A}^*)$ such that $\|\rho\|_{\mathcal{A}^*} = 1$ and $\rho(F) \geq 0$ for all $F \in \mathcal{A}$ satisfying $F \geq 0$. And put

$$\mathfrak{S}^m(\mathcal{A}^*) = \{\rho \in \mathcal{A}^* \mid \rho \text{ is a mixed state}\}. \quad (1)$$

A mixed state $\rho (\in \mathfrak{S}^m(\mathcal{A}^*))$ is called a *pure state* if it satisfies that “ $\rho = \theta \rho_1 + (1 - \theta) \rho_2$ for some $\rho_1, \rho_2 \in \mathfrak{S}^m(\mathcal{A}^*)$ and $0 < \theta < 1$ ” implies “ $\rho = \rho_1 = \rho_2$ ”. Put

$$\mathfrak{S}^p(\mathcal{A}^*) = \{\rho \in \mathfrak{S}^m(\mathcal{A}^*) \mid \rho \text{ is a pure state}\}, \quad (2)$$

which is called a *state space*. The Riesz theorem (cf. [14]) says that $C_0(\Omega)^* = \mathcal{M}(\Omega) = \{\rho \mid \rho \text{ is a signed measure on } \Omega\}$, $\mathfrak{S}^m(C_0(\Omega)^*) = \mathcal{M}_{+1}^m(\Omega) = \{\rho \mid \rho \text{ is a measure on } \Omega \text{ such that } \rho(\Omega) = 1\}$. Also, it is well known (cf. [12]) that $\mathfrak{S}^p(B_c(H)^*) = \{|u\rangle\langle u| \text{ (i.e., the Dirac notation)} \mid \|u\|_H = 1\}$, and $\mathfrak{S}^p(C_0(\Omega)^*) = \mathcal{M}_{+1}^p(\Omega) = \{\delta_{\omega_0} \mid \delta_{\omega_0} \text{ is a point measure at } \omega_0 \in \Omega\}$, where $\int_{\Omega} f(\omega) \delta_{\omega_0}(d\omega) = f(\omega_0)$

($\forall f \in C_0(\Omega)$). The latter implies that $\mathfrak{S}^p(C_0(\Omega)^*)$ can be also identified with Ω (called a *spectrum space* or simply *spectrum*) such as

$$\mathfrak{S}^p(C_0(\Omega)^*) \ni \delta_\omega \leftrightarrow \omega \in \underset{\text{(state space)}}{\Omega} \underset{\text{(spectrum)}}{\quad} \quad (3)$$

Here, assume that the C^* -algebra $\mathcal{A}(\subseteq B(H))$ is unital, i.e., it has the identity I . This assumption is not unnatural, since, if $I \notin \mathcal{A}$, it suffices to reconstruct the \mathcal{A} such that it includes $\mathcal{A} \cup \{I\}$. In this sense, the $C_0(\Omega)$ is often denoted by $C(\Omega)$.

According to the noted idea (cf. [15]) in quantum mechanics, an *observable* $O \equiv (X, \mathcal{F}, F)$ in \mathcal{A} is defined as follows:

- (D₁) [Field] X is a set, $\mathcal{F}(\subseteq 2^X$, the power set of X) is a field of X , that is, “ $\Xi_1, \Xi_2 \in \mathcal{F} \Rightarrow \Xi_1 \cup \Xi_2 \in \mathcal{F}$ ”, “ $\Xi \in \mathcal{F} \Rightarrow X \setminus \Xi \in \mathcal{F}$ ”.
- (D₂) [Countably additivity] F is a mapping from \mathcal{F} to \mathcal{A} satisfying: (a): for every $\Xi \in \mathcal{F}$, $F(\Xi)$ is a non-negative element in \mathcal{A} such that $0 \leq F(\Xi) \leq I$, (b): $F(\emptyset) = 0$ and $F(X) = I$, where 0 and I is the 0-element and the identity in \mathcal{A} respectively. (c): for any countable decomposition $\{\Xi_1, \Xi_2, \dots\}$ of $\Xi \in \mathcal{F}$ (i.e., $\Xi_k, \Xi \in \mathcal{F}$ such that $\bigcup_{k=1}^\infty \Xi_k = \Xi$, $\Xi_i \cap \Xi_j = \emptyset (i \neq j)$), it holds that

$$\lim_{K \rightarrow \infty} \rho(F(\bigcup_{k=1}^K \Xi_k)) = \rho(F(\Xi)) \quad (\forall \rho \in \mathfrak{S}^m(\mathcal{A}^*))$$

(i.e., in the sense of weak convergence). (4)

Remark 1. By the Hopf extension theorem (cf. [14]), we have the mathematical probability space $(X, \overline{\mathcal{F}}, \rho^m(F(\cdot)))$ where $\overline{\mathcal{F}}$ is the smallest σ -field such that $\mathcal{F} \subseteq \overline{\mathcal{F}}$. For the other formulation (i.e., W^* -algebraic formulation), see, for example, the appendix in [4].

2.3. Pure measurement theory in (A₁)

In what follows, we shall explain PMT in (A₁).

With any *system* S , a C^* -algebra $\mathcal{A}(\subseteq B(H))$ can be associated in which the pure measurement theory (A₁) of that system can be formulated. A *state* of the system S is represented by an element $\rho(\in \mathfrak{S}^p(\mathcal{A}^*))$ and an *observable* is represented by an observable $O = (X, \mathcal{F}, F)$ in \mathcal{A} . Also, the *measurement of the observable* O for the system S with the state ρ is denoted by $M_{\mathcal{A}}(O, S_{[\rho]})$ (or more precisely, $M_{\mathcal{A}}(O := (X, \mathcal{F}, F), S_{[\rho]})$). An observer can

obtain a measured value $x (\in X)$ by the measurement $M_{\mathcal{A}}(O, S_{[\rho]})$.

The Axiom^P 1 presented below is a kind of mathematical generalization of Born’s probabilistic interpretation of quantum mechanics. And thus, it is a statement without reality.

Axiom^P 1 [Pure Measurement]. *The probability that a measured value $x (\in X)$ obtained by the measurement $M_{\mathcal{A}}(O := (X, \mathcal{F}, F), S_{[\rho_0]})$ belongs to a set $\Xi (\in \mathcal{F})$ is given by $\rho_0(F(\Xi))$.*

Next, we explain Axiom 2 in (A). Let (T, \leq) be a tree, i.e., a partial ordered set such that “ $t_1 \leq t_3$ and $t_2 \leq t_3$ ” implies “ $t_1 \leq t_2$ or $t_2 \leq t_1$ ”. In this paper, we assume that T is finite. Assume that there exists an element $t_0 \in T$, called the *root* of T , such that $t_0 \leq t (\forall t \in T)$ holds. Put $T_{\leq}^2 = \{(t_1, t_2) \in T^2 \mid t_1 \leq t_2\}$. The family $\{\Phi_{t_1, t_2} : \mathcal{A}_{t_2} \rightarrow \mathcal{A}_{t_1}\}_{(t_1, t_2) \in T_{\leq}^2}$ is called a *causal relation (due to the Heisenberg picture)*, if it satisfies the following conditions (E₁) and (E₂).

(E₁) With each $t \in T$, a C^* -algebra \mathcal{A}_t is associated.

(E₂) For every $(t_1, t_2) \in T_{\leq}^2$, a Markov operator $\Phi_{t_1, t_2} : \mathcal{A}_{t_2} \rightarrow \mathcal{A}_{t_1}$ is defined (i.e., $\Phi_{t_1, t_2} \geq 0$, $\Phi_{t_1, t_2}(I_{\mathcal{A}_{t_2}}) = I_{\mathcal{A}_{t_1}}$). And it satisfies that $\Phi_{t_1, t_2} \Phi_{t_2, t_3} = \Phi_{t_1, t_3}$ holds for any $(t_1, t_2), (t_2, t_3) \in T_{\leq}^2$.

The family of dual operators $\{\Phi_{t_1, t_2}^* : \mathfrak{S}^m(\mathcal{A}_{t_1}^*) \rightarrow \mathfrak{S}^m(\mathcal{A}_{t_2}^*)\}_{(t_1, t_2) \in T_{\leq}^2}$ is called a *dual causal relation (due to the Schrödinger picture)*. When $\Phi_{t_1, t_2}^* (\mathfrak{S}^p(\mathcal{A}_{t_1}^*)) \subseteq (\mathfrak{S}^p(\mathcal{A}_{t_2}^*))$ holds for any $(t_1, t_2) \in T_{\leq}^2$, the causal relation is said to be deterministic.

Now Axiom 2 in the measurement theory (A) is presented as follows:

Axiom 2 [Causality]. *The causality is represented by a causal relation $\{\Phi_{t_1, t_2} : \mathcal{A}_{t_2} \rightarrow \mathcal{A}_{t_1}\}_{(t_1, t_2) \in T_{\leq}^2}$.*

2.4. Statistical measurement theory in (A₂)

We shall introduce the following notation: It is usual to consider that we do not know the pure state $\rho_0^p (\in \mathfrak{S}^p(\mathcal{A}^*))$ when we take a measurement $M_{\mathcal{A}}(O, S_{[\rho_0^p]})$. That is because we usually take a measurement $M_{\mathcal{A}}(O, S_{[\rho_0^p]})$ in order to know the state ρ_0^p . Thus, when we want to emphasize that we do not know the state ρ_0^p , $M_{\mathcal{A}}(O, S_{[\rho_0^p]})$ is denoted by $M_{\mathcal{A}}(O, S_{[*]})$. Also, when we know the distribution $\rho_0^m (\in \mathfrak{S}^m(\mathcal{A}^*))$ of the unknown state ρ_0^p , the $M_{\mathcal{A}}(O, S_{[\rho_0^p]})$ is denoted by $M_{\mathcal{A}}(O, S_{[*]}(\{\rho_0^m\}))$. The ρ_0^m and $M_{\mathcal{A}}(O, S_{[*]}(\{\rho_0^m\}))$ is respectively called a mixed state and a statistical measurement.

The Axiom^S 1 presented below is a kind of mathematical generalization of Axiom^P 1.

Axiom^S 1 [Statistical measurement]. *The probability that a measured value x ($\in X$) obtained by the statistical measurement $M_{\mathcal{A}}(O \equiv (X, \mathcal{F}, F), S_{[\ast]}(\{\rho_0^m\}))$ belongs to a set $\Xi(\in \mathcal{F})$ is given by $\rho_0^m(F(\Xi))$ ($= \mathcal{A}^*(\rho_0^m, F(\Xi))_{\mathcal{A}}$).*

Here, Axiom 2 and Interpretation (H) mentioned in the following section are common in (A₁) and (A₂).

3. The Linguistic Interpretation (Figure 1:②,③,⑥)

According to [4], we shall explain the linguistic interpretation of quantum mechanics. The measurement theory (A) asserts

- (F) Obey Axioms 1 and 2. And, describe any ordinary phenomenon according to Axioms 1 and 2 (in spite that Axioms 1 and 2 can not be tested experimentally).

Still, most readers may be perplexed how to use Axioms 1 and 2 since there are various usages. Thus, the following problem is significant.

- (G) How should Axioms 1 and 2 be used?

Note that reality is not reliable since Axioms 1 and 2 are statements without reality.

Here, in spite of the linguistic turn (Figure 1:③) and the mathematical generalization from $B(H)$ to a C^* -algebra \mathcal{A} , we consider that the dualism (i.e., the spirit of so called Copenhagen interpretation) of quantum mechanics is inherited to measurement theory (Figure 1:②, or also, see (I) later). Thus, we present the following interpretation (H) [(H₁)–(H₃)]. That is, as the answer to the question (G), we propose:

- (H₁) Consider the dualism composed of “observer” and “system(=measuring object)”. And therefore, “observer” and “system” must be absolutely separated.
- (H₂) Only one measurement is permitted. And thus, the state after a measurement is meaningless since it can not be measured any longer. Also, the causality should be assumed only in the side of system, however, a state never moves. Thus, the Heisenberg picture should be adopted.
- (H₃) Also, the observer does not have the space-time. Thus, the question: “When and where is a measured value obtained?” is out of measurement theory,

and so on.

Although N. Bohr (i.e., the chief proponent of the Copenhagen interpretation) said, in the Bohr–Einstein debates [3, 16], that the interpretation of a physical theory has to rely on an experimental practice. However, we consider that all confusion is due to the preconception that the Copenhagen interpretation is within physics. In this sense, we agree with A. Einstein, who never accepted the Copenhagen interpretation as physics. That is, in spite of Bohr’s realistic view, we propose the following linguistic world view (Figure 1:③):

- (I) In the beginning was the language called measurement theory (with the interpretation (H)). And, for example, quantum mechanics can be fortunately described in this language. And moreover, almost all scientists have already mastered this language partially and informally since statistics (at least, its basic part) is characterized as one of aspects of measurement theory (cf. [4–11]).

In this sense, we consider that measurement theory holds as a kind of language-game (with the rule (Axioms 1 and 2, Interpretation (H))), and therefore, measurement theory is regarded as the axiomatization (Figure 1:⑥) of the philosophy of language (i.e., Saussure’s linguistic world view).

4. How to Use the Linguistic Interpretation (H)

4.1. Parallel measurement, the law of large numbers

For each $k = 1, 2, \dots, K$, consider a measurement $M_{\mathcal{A}_k}(O_k := (X_k, \mathcal{F}_k, F_k), S_{[\rho_k]})$. However, the interpretation (H₂) says that only one measurement is permitted. Thus, we consider the spatial tensor C^* -algebra $\bigotimes_{k=1}^K \mathcal{A}_k (\subseteq B(\bigotimes_{k=1}^K H_k))$, and consider the product space $\times_{k=1}^K X_k$ and the product field $\boxtimes_{k=1}^K \mathcal{F}_k$, which is defined by the smallest field that contains a family $\{\times_{k=1}^K \Xi_k \mid \Xi_k \in \mathcal{F}_k, k = 1, 2, \dots, K\}$. Define the parallel observable $\bigotimes_{k=1}^K O_k = (\times_{k=1}^K X_k, \boxtimes_{k=1}^K \mathcal{F}_k, \tilde{F})$ in the tensor C^* -algebra $\bigotimes_{k=1}^K \mathcal{A}_k$ such that

$$\begin{aligned} \tilde{F}(\times_{k=1}^K \Xi_k) &= \bigotimes_{k=1}^K F_k(\Xi_k) \\ (\forall \Xi_k \in \mathcal{F}_k, k = 1, 2, \dots, K). \end{aligned}$$

Then, the above $\{M_{\mathcal{A}_k}(O_k, S_{[\rho_k]})\}_{k=1}^K$ is represented

by the parallel measurement $M_{\otimes_{k=1}^K \mathcal{A}_k}(\otimes_{k=1}^K O_k, S_{[\otimes_{k=1}^K \rho_k]})$, which is also denoted by $\otimes_{k=1}^K M_{\mathcal{A}_k}(O_k, S_{[\rho_k]})$. Consider a particular case such that, $\mathcal{A} = \mathcal{A}_k$, $O = (X, \mathcal{F}, F) = (X_k, \mathcal{F}_k, F_k)$, $\rho = \rho_k$ ($\forall k = 1, 2, \dots, K$). Let $(x_1, x_2, \dots, x_K) \in X^K$ be a measured value by the parallel measurement $\otimes_{k=1}^K M_{\mathcal{A}}(O, S_{[\rho]})$. Then, using Axiom 1, we see the law of large numbers, that is, for sufficiently large K ,

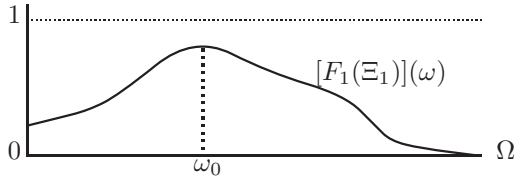
$$\rho(F(\Xi)) \approx \frac{\#\{k = 1, 2, \dots, K \mid x_k \in \Xi\}}{K} \quad (\forall \Xi \in \mathcal{F})$$

holds, where $\#[A]$ is the the number of elements of the set A . This is, of course, most fundamental in science. Also, this is the reason that the term “probability” is used in Axiom 1.

4.2. Maximal likelihood estimation

Consider the classical cases (i.e., $\mathcal{A} = C_0(\Omega)$). It may be usual to consider that Axiom 1 leads the following statement, i.e., maximum likelihood estimation in classical measurements:

- (J) [Maximum likelihood estimation in classical PMT]; When we know that a measured value obtained by a measurement $M_{C_0(\Omega)}(O_1 := (X_1, \mathcal{F}_1, F_1), S_{[*]})$ belongs to $\Xi_1 (\in \mathcal{F}_1)$, there is a reason to infer that the unknown state $[*] = \delta_{\omega_0} (\in \mathfrak{S}^p(C_0(\Omega)^*) \approx \Omega, \text{ by (3)})$ where $\omega_0 (\in \Omega)$ is defined by $[F_1(\Xi_1)](\omega_0) = \max_{\omega \in \Omega} [F_1(\Xi_1)](\omega)$ if it exists.



Although this (J) is surely handy, note that the (H₂) says that it is illegal to regard the ρ_0 as the state after the measurement $M_{C_0(\Omega)}(O_1, S_{[*]})$. Thus, strictly speaking, the (J) is informal.

By a similar method as the lead of the (J), we can easily see the following statement (K), which should be regarded as the measurement theoretical form of maximum likelihood estimation (*cf.* [6], or Corollary 5.5 in [10]).

- (K) [Maximum likelihood estimation in general PMT]; When we know that a measured value obtained by a measurement $M_{\mathcal{A}}(O := (X_1 \times X_2, \mathcal{F}_1 \boxtimes \mathcal{F}_2, F), S_{[*]})$ belongs to $\Xi_1 \times X_2$, there is a reason to infer that

the probability that the measured value belongs to $\Xi_1 \times \Xi_2$ ($\forall \Xi_2 \in \mathcal{F}_2$) is given by the following conditional probability:

$$\frac{\rho_0(F(\Xi_1 \times \Xi_2))}{\rho_0(F(\Xi_1 \times X_2))} \quad (5)$$

where $\rho_0 (\in \mathfrak{S}^p(\mathcal{A}^*))$ is defined by $\rho_0(F(\Xi_1 \times X_2)) = \max_{\rho \in \mathfrak{S}^p(\mathcal{A}^*)} \rho(F(\Xi_1 \times X_2))$ if it exists. Here, note that the ρ_0 is not the state after the measurement $M_{\mathcal{A}}(O, S_{[*]})$.

This (K), which also includes quantum cases, is most fundamental in statistics, and thus, we believe (*cf.* [6,9]) that statistics is one of aspects of measurement theory. If it be so, we can answer to the problem (B₂). For the relation between the informal (J) and the formal (K), see Remark 2 later.

4.3. Simultaneous measurement

For each $k = 1, 2, \dots, K$, consider a measurement $M_{\mathcal{A}}(O_k := (X_k, \mathcal{F}_k, F_k), S_{[\rho]})$. However, since the (H₂) says that only one measurement is permitted, the measurements $\{M_{\mathcal{A}}(O_k, S_{[\rho]})\}_{k=1}^K$ should be reconsidered in what follows. Under the commutativity condition such that

$$F_i(\Xi_i)F_j(\Xi_j) = F_j(\Xi_j)F_i(\Xi_i) \quad (6)$$

$$(\forall \Xi_i \in \mathcal{F}_i, \forall \Xi_j \in \mathcal{F}_j, i \neq j),$$

we can define the product observable $\times_{k=1}^K O_k = (\times_{k=1}^K X_k, \boxtimes_{k=1}^K \mathcal{F}_k, \times_{k=1}^K F_k)$ in \mathcal{A} such that

$$(\times_{k=1}^K F_k)(\times_{k=1}^K \Xi_k) = F_1(\Xi_1)F_2(\Xi_2) \cdots F_K(\Xi_K)$$

$$(\forall \Xi_k \in \mathcal{F}_k, \forall k = 1, \dots, K).$$

Here, $\boxtimes_{k=1}^K \mathcal{F}_k$ is the smallest field including the family $\{\times_{k=1}^K \Xi_k : \Xi_k \in \mathcal{F}_k, k = 1, 2, \dots, K\}$. Then, the above $\{M_{\mathcal{A}}(O_k, S_{[\rho]})\}_{k=1}^K$ is, under the commutativity condition (6), represented by the simultaneous measurement $M_{\mathcal{A}}(\times_{k=1}^K O_k, S_{[\rho]})$.

Remark 2. [The relation between (J) and (K)] Consider the (K) in the classical cases, i.e., $\mathcal{A} = C_0(\Omega)$. And assume the simultaneous observable $F = F_1 \times F_2$ in (5). Then, putting $\rho_0 = \delta_{\omega_0}$ (i.e., the point measure at ω_0), we see that

$$(5) = \frac{[F_1(\Xi_1) \times F_2(\Xi_2)](\omega_0)}{[F_1(\Xi_1) \times F_2(X_2)](\omega_0)} = [F_2(\Xi_2)](\omega_0)$$

$$= \rho_0(F_2(\Xi_2)).$$

Since this equality holds for any $O_2 = (X_2, \mathcal{F}_2, F_2)$ and any $\Xi_2 \in \mathcal{F}_2$, some may want to regard the ρ_0 as the state after the measurement $M_{C_0(\Omega)}(O_1 := (X_1, \mathcal{F}_1, F_1), S_{[*]})$ in the (J). Thus, in spite of the (H₂), the (J) may be allowed in classical cases if the ρ_0 may be regarded as something represented by the term such as “imaginary state”. This is the meaning of the informal (J).

4.4. Sequential causal observable and its realization

Consider a tree $(T \equiv \{t_0, t_1, \dots, t_n\}, \leq)$ with the root t_0 . This is also characterized by the map $\pi : T \setminus \{t_0\} \rightarrow T$ such that $\pi(t) = \max\{s \in T \mid s < t\}$. Let $\{\Phi_{t,t'} : \mathcal{A}_{t'} \rightarrow \mathcal{A}_t\}_{(t,t') \in T_{\leq}^2}$ be a causal relation, which is also represented by $\{\Phi_{\pi(t),t} : \mathcal{A}_t \rightarrow \mathcal{A}_{\pi(t)}\}_{t \in T \setminus \{t_0\}}$. Let an observable $O_t \equiv (X_t, \mathcal{F}_t, F_t)$ in the \mathcal{A}_t be given for each $t \in T$. Note that $\Phi_{\pi(t),t} O_t$ ($\equiv (X_t, \mathcal{F}_t, \Phi_{\pi(t),t} F_t)$) is an observable in the $\mathcal{A}_{\pi(t)}$.

The pair $[\mathbb{O}_T] = [\{O_t\}_{t \in T}, \{\Phi_{t,t'} : \mathcal{A}_{t'} \rightarrow \mathcal{A}_t\}_{(t,t') \in T_{\leq}^2}]$ is called a *sequential causal observable*. For each $s \in T$, put $T_s = \{t \in T \mid t \geq s\}$. And define the observable $\hat{O}_s \equiv (\times_{t \in T_s} X_t, \boxtimes_{t \in T_s} \mathcal{F}_t, \hat{F}_s)$ in \mathcal{A}_s as follows:

$$\hat{O}_s = \begin{cases} O_s & (\text{if } s \in T \setminus \pi(T)) \\ O_s \times (\times_{t \in \pi^{-1}(\{s\})} \Phi_{\pi(t),t} \hat{O}_t) & (\text{if } s \in \pi(T)) \end{cases} \quad (7)$$

if the commutativity condition holds (i.e., if the product observable $O_s \times (\times_{t \in \pi^{-1}(\{s\})} \Phi_{\pi(t),t} \hat{O}_t)$ exists) for each $s \in \pi(T)$. Using (7) iteratively, we can finally obtain the observable \hat{O}_{t_0} in \mathcal{A}_{t_0} . The \hat{O}_{t_0} is called the realization (or, realized causal observable) of $[\mathbb{O}_T]$.

Remark 3. [Kolmogorov extension theorem] In the general cases such that countable additivity and infinite T are required, the existence of the above \hat{O}_{t_0} is, by using the Kolmogorov extension theorem in probability theory [17], proved in the W^* -algebraic formulation (cf. [10]). We think that this fact is evidence that the interpretation (H₂) is hidden behind the utility of the Kolmogorov extension theorem. Recall the following well-known statement that always appears in the beginning of probability theory:

(L) Let (X, \mathcal{F}, P) be a probability space. Then, the probability that an event $\Xi (\in \mathcal{F})$ occurs is given by $P(\Xi)$,

which, as well as Axiom 1, is a statement without reality. We consider that the Kolmogorov extension theorem is regarded as one of the finest answers to the problem: “How should the statement (L) be used?”. That is, in mathematical probability theory, the answer is presented as the form of a basic theorem (i.e., the Kolmogorov extension theorem). On the other hand, in measurement theory, the problem (G) is answered by the interpretation (H).

Remark 4. [Wavefunction collapse] Again reconsider the (K) in the simplest case that $T = \{t_0, t_1\}, \pi(t_1) = t_0$. Taking a measurement $M_{\mathcal{A}_{t_0}}(O_{t_0}, S_{[\rho_0]})$, we know that the measured value belongs to $\Xi_0 (\in \mathcal{F}_{t_0})$. Then, it may be usual to consider that a certain wavefunction collapse happens by the measurement, that is, $\mathfrak{S}^p(\mathcal{A}_{t_0}^*) \ni \rho_0 \mapsto \rho_0^{\Xi_0} \in \mathfrak{S}^p(\mathcal{A}_{t_0}^*)$. And continuously, we take a measurement $M_{\mathcal{A}_{t_0}}(\Phi_{t_0,t_1} O_{t_1}, S_{[\rho_0^{\Xi_0}]})$. Here, the probability that a measured value belongs to $\Xi_1 (\in \mathcal{F}_{t_1})$ is, by Axiom 1, given by $\rho_0^{\Xi_0}(\Phi_{t_0,t_1} F_1(\Xi_1))$. However, this $\rho_0^{\Xi_0}(\Phi_{t_0,t_1} F_1(\Xi_1))$ must be equal to the conditional probability

$$\frac{\rho_0(\hat{F}(\Xi_0 \times \Xi_1))}{\rho_0(\hat{F}(\Xi_0 \times X_1))}$$

if the commutativity condition holds (i.e., the simultaneous observable $\hat{O}_{t_0} = O_{t_0} \times \Phi_{t_0,t_1} O_{t_1} = (X_{t_0} \times X_{t_1}, \mathcal{F}_{t_0} \boxtimes \mathcal{F}_{t_1}, \hat{F} := F_{t_0} \times \Phi_{t_0,t_1} F_{t_1})$ exists). This implies that it suffices to consider only the measurement $M_{\mathcal{A}_{t_0}}(O_{t_0} \times \Phi_{t_0,t_1} O_{t_1}, S_{[\rho_0]})$. That is, two measurements $M_{\mathcal{A}_{t_0}}(O_{t_0}, S_{[\rho_0]})$ and $M_{\mathcal{A}_{t_0}}(\Phi_{t_0,t_1} O_{t_1}, S_{[\rho_0^{\Xi_0}]})$ are not needed. Also, if the commutativity condition is ignored in the argument of the wavefunction collapse, it is doubtful.

4.5. Bayes’ method in classical SMT

Let $O_1 \equiv (X, \mathcal{F}, F)$ be an observable in a commutative C^* -algebra $C(\Omega)$. And let $O_2 \equiv (Y, \mathcal{G}, G)$ be any observable in $C(\Omega)$. Consider the product observable $O_1 \times O_2 \equiv (X \times Y, \mathcal{F} \boxtimes \mathcal{G}, F \times G)$ in $C(\Omega)$.

Assume that we know that the measured value (x, y) obtained by a simultaneous measurement $M_{C(\Omega)}(O_1 \times O_2, S_{[*]}(\{\nu_0\}))$ belongs to $\Xi \times Y (\in \mathcal{F} \boxtimes \mathcal{G})$. Then, by Axiom^S 1, we can infer that

(M₁) the probability $P_{\Xi}(G(\Gamma))$ that y belongs to $\Gamma (\in \mathcal{G})$ is given by

$$P_{\Xi}(G(\Gamma)) = \frac{\int_{\Omega} [F(\Xi)G(\Gamma)](\omega) \nu_0(d\omega)}{\int_{\Omega} [F(\Xi)](\omega) \nu_0(d\omega)} \quad (\forall \Gamma \in \mathcal{G}).$$

Thus, we can assert that:

(M₂) [Bayes' method, cf. [6, 9]]. When we know that a measured value obtained by a measurement $M_{C(\Omega)}(O_1 \equiv (X, \mathcal{F}, F), S_{[*]}(\{\nu_0\}))$ belongs to Ξ , there is a reason to infer that the mixed state after the measurement is equal to ν_0^a ($\in \mathcal{M}_{+1}^m(\Omega)$), where

$$\nu_0^a(D) = \frac{\int_D [F(\Xi)](\omega) \nu_0(d\omega)}{\int_\Omega [F(\Xi)](\omega) \nu_0(d\omega)} \quad (\forall D \in \mathcal{B}_\Omega).$$

This (as well as (J)) is, of course, informal.

4.6. Heisenberg's uncertainty principle

In this section, we use the W^* -algebraic formulation, that is, we consider the basic structure $[B_c(H), B(H)]_{B(H)}$ (cf. Section 5.5 later). Let A_1 and A_2 be self-adjoint operators on a Hilbert space H . Note that the spectral representation $O_k = (\mathbb{R}, \mathcal{B}_\mathbb{R}, E_{A_k})$ of A_k is regarded as the observable in $B(H)$. Thus, we have two measurements $M_{B(H)}(O_k, S_{[\rho_u]})$ ($\rho_u = |u\rangle\langle u| \in \mathfrak{S}^p(B_c(H)^*), k = 1, 2$). However, since O_1 and O_2 do not generally commute, Interpretation (H₂) says that it is impossible to measure $M_{B(H)}(O_1, S_{[\rho]})$ and $M_{B(H)}(O_2, S_{[\rho]})$ simultaneously. Then we consider another Hilbert space K , $\rho_s = |s\rangle\langle s| \in \mathfrak{S}^p(B_c(K)^*)$, self-adjoint operators \hat{A}_k on a tensor Hilbert space $H \otimes K$ such that

$$(N_1) : \langle v, A_k v \rangle_H = \langle v \otimes s, \hat{A}_k v \otimes s \rangle_{H \otimes K} \quad (\forall v \in H)$$

$$(N_2) : \hat{A}_1 \text{ and } \hat{A}_2 \text{ commute}$$

The existence is assured (cf. Theorem 1 in [1]). Define the observable $\hat{O}_k = (\mathbb{R}, \mathcal{B}_\mathbb{R}, E_{\hat{A}_k})$ in $B(H \otimes K)$ by the spectral representation of \hat{A}_k . By the commutativity (N₂), we get the product observable $\hat{O}_1 \times \hat{O}_2$, which is called the approximately simultaneous observable of A_1 and A_2 . Thus we can take an approximately simultaneous measurement $M_{B(H \otimes K)}(\hat{O}_1 \times \hat{O}_2, S_{[\rho_u \otimes \rho_s]})$ of A_1 and A_2 . Putting $\Delta_k = \|(\hat{A}_k - A_k \otimes I)(u \otimes s)\|_{H \otimes K}$, we have the following Heisenberg's uncertainty principle (cf. Theorem 2 in [1]):

$$\Delta_1 \cdot \Delta_2 \geq \frac{1}{2} |\langle A_1 u, A_2 u \rangle_H - \langle A_2 u, A_1 u \rangle_H| \quad (\forall u \in H) \quad (8)$$

This should not be confused with Robertson's uncertainty principle [2], which says

$$\sigma_1 \cdot \sigma_2 \geq \frac{1}{2} |\langle A_1 u, A_2 u \rangle_H - \langle A_2 u, A_1 u \rangle_H| \quad (\forall u \in H) \quad (9)$$

where σ_k is defined by $\|(A_k - \langle u, A_k u \rangle)u\|_H$.

5. Traditional Philosophies (Figure 1)

According to [5], we shall explain **Figure 1**: [①-⑧] from the measurement theoretical point of view in what follows..

5.1. Realistic world view and Linguistic world view

Figure 1 says that the realistic world view ⑧ and the linguistic world view ① exist together in science. Some may ask:

(O) Why is the series ① (or, idealism originated by Plato) underestimated in science?

We think that the reason is due to the fact that the ① is lacking in the axiomatization ⑥ if we do not have measurement theory. That is, we believe that there is no scientific world view without axiomatization.

5.2. Dualism

Interpretation (H₁) says “Image **Figure 2** whenever measurement theory is used”.

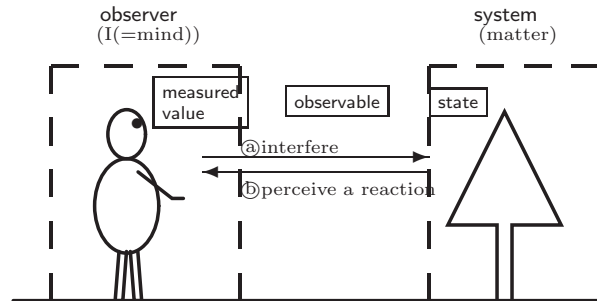


Figure 2. Descartes' figure in MT

where the interaction [a] and [b] must not be emphasized, that is, it must be implicit. That is because, if it is explicitly stated, the dualism (H₁) is violated.

John Locke's famous sayings “primary quality (e.g., length, weight, etc.)” and “secondary quality

(e.g., sweet, dark, cold, etc.)” urge us to associate the following correspondence:

state \leftrightarrow primary quality, observable \leftrightarrow second quality

Also, it may be understandable to regard “observable” as “measurement instrument” or “sensory system”, for example, eyes, glasses, condensation trail, etc. And further, it is natural to consider that there is no measured value without observer’s brain (i.e., when something reaches observer’s brain, it becomes a measured value). Thus, we want to consider the following correspondence in **Table 1**.

Table 1: Descartes vs. MT

Descartes	mind (brain)	body	matter
MT	measured value	observable	state

In the history of philosophy, two kinds of dualisms (based on “mind-body dualism” and “matter-mind dualism”) may be frequently discussed. However, it should be noted that the dualism (H_1) is composed of three concepts as mentioned in **Table 1**. Also, the following question is nonsense in the linguistic world view.

- (P₁) What is “measured value” (or, observable, state)? Or equivalently, in the sense of **Table 1**, what is “mind” (or, body, matter)? And moreover, what is “probability” (or, causality)?

Therefore, we must admit that the correspondence in **Table 1** is rather figurative, that is, it is not worth discussing the problem (P₁) seriously in the linguistic world view. From the great history of philosophy, we learned that the serious consideration (i.e., the consideration from the realistic world view) of the problem (P₁) (e.g., mind-body problem, etc.) always led us into blind alleys.

We have to confirm that we are now in the side of the linguistic world view (i.e., after the linguistic turn ③ in **Figure 1**) and not the realistic world view. Thus, our interest always focuses on the problem:

- (P₂) How should the term: “measured value” (or, observable, state) be used? And moreover, How should “probability” (or, causality) be described?

We, of course, assert that this answer is just given by measurement theory (i.e., Axioms 1 and 2, Interpretation (H)). After accepting measurement theory,

what we can do in measurement theory is only to trust in man’s linguistic competence. This is our linguistic world view (I).

Here, we want to consider that the following two are essentially the same:

- (Q₁) *To be is to be perceived (by Berkeley)*
 (Q₂) *There is no science without measurement (particularly, measured value).*

Also, in the sense of **Table 1**, these are similar to

- (Q₃) *There is no science without human’s brain,*

which may be also similar to Kant’s assertion (see Section 5.7 later).

5.3. I think, therefore I am.

Figure 2 (Descartes’ figure in MT) may be inspired from the Descartes primary principle:

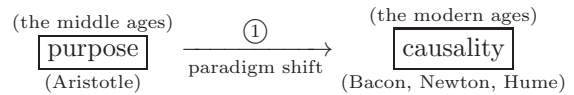
- (R) *I think, therefore I am.*

However, it should be noted that the statement (R) is not a statement in the dualism of Interpretation (H_1). That is because it is natural to assume that “I”=“observer” and “T”=“system” in the (R), which clearly contradicts the (H_1). We may see an irony in the fact that the non-dualistic statement (R) gives foundations to the dualistic **Figure 2**. However, it is sure that the establishment of “I” in (R) brought us modern science (**Figure 1:④**).

Also, it is natural to consider that Heidegger’s saying: “In-der-Welt-sein” is out of **Figure 2**, and thus, out of measure theory. If some succeed the axiomatization of “In-der-Welt-sein”, it will be the powerful rival of measurement theory in science.

5.4. Causality and Probability

The following paradigm shift (*cf.* **Figure 1:①**):



is the greatest paradigm shift throughout all history of science. However, it should be noted that there are several ideas for “causality”. For example Newton’s causality is realistic, and Hume’s causality is subjective. On the other hand, our causality (i.e., Axiom 2) is linguistic.

Although some philosophers (e.g., K. Popper [19]) consider that the discovery of “probability” is as

great as that of “causality”, it is sure that the former is underestimated in science. The reason of the underestimation may be due to the fact that the “probability” is never presented in a certain world view (but in mathematics (*cf.* [17]), on the other hand, the “causality” is established in the world view (i.e., Newtonian mechanics). We think that it is desirable to understand the two concepts (i.e., “probability” and “causality”) in the same world view. It should be noted that this is realized in Axioms 1 and 2 of measurement theory.

5.5. Leibniz-Clarke Correspondence (Space-Time Problem)

In this section, first we must prepare the term “spectrum” in the formula (3). Consider the pair $[\mathcal{A}, \overline{\mathcal{A}}]_{B(H)}$, called a *basic structure* (*cf.* Appendix in [4]). Here, $\mathcal{A}(\subseteq B(H))$ is a C^* -algebra, and $\overline{\mathcal{A}}$ ($\mathcal{A} \subseteq \overline{\mathcal{A}} \subseteq B(H)$) is a particular C^* -algebra (called a W^* -algebra) such that $\overline{\mathcal{A}}$ is the weak closure of \mathcal{A} in $B(H)$. Let $\mathcal{A}_S (\subseteq \overline{\mathcal{A}})$ be the commutative C^* -subalgebra. Note that \mathcal{A}_S is represented such that $\mathcal{A}_S = C_0(\Omega_S)$ for some locally compact Hausdorff space Ω_S (*cf.* [12]). The Ω_S is called a *spectrum*. For example, consider one particle quantum system, formulated in a basic structure $[B_c(L^2(\mathbb{R}^3)), B(L^2(\mathbb{R}^3))]_{B(L^2(\mathbb{R}^3))}$. Then, we can choose the commutative C^* -algebra $C_0(\mathbb{R}^3) (\subset B(L^2(\mathbb{R}^3)))$, and thus, we get the spectrum \mathbb{R}^3 . This simple example will make us proposal (S₂) later.

In Leibniz-Clarke correspondence (1715–1716), they (i.e., Leibniz and Clarke(=Newton’s friend)) discussed “space-time problem”. Their ideas are summarized as follows:

$$(S_1) \left\{ \begin{array}{ll} \textcircled{R}: \text{Newton, Clarke} \cdots \begin{array}{c} \text{(space-time in physics)} \\ \text{realistic space-time} \end{array} \\ \text{(realistic world view)} \\ \textcircled{L}: \text{Leibniz} \cdots \begin{array}{c} \text{(space-time in language)} \\ \text{linguistic space-time} \end{array} \\ \text{(linguistic world view)} \end{array} \right.$$

That is, Newton considered “What is space-time?”. On the other hand, Leibniz considered “How should space-time be represented?”, though he did not propose his language. Measurement theory is in Leibniz’s side, and asserts that

- (S₂) Space should be described as a kind of spectrum. And time should be described as a kind of tree. In other words, time is represented by a parameter t in a linear ordered tree T .

Therefore, we think that the Leibniz-Clarke debates should be essentially regarded as “the linguistic world view \textcircled{L} ” vs. “the realistic world view \textcircled{R} ”. Hence, the statement (S₂) should be added to Interpretation (H) as sub-interpretation of measurement theory.

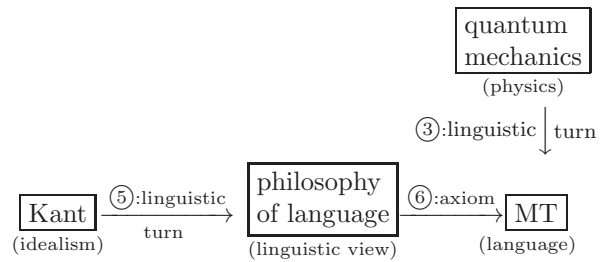
5.6. Observer’s time

It is usual to consider that quantum mechanics and observer’s time are incompatible. This leads Interpretation (H₃), which says that observer’s time is nonsense in measurement theory. That is, there is no tense — past, present, future — in measurement theory, and therefore, in science. Many philosophers (e.g., Augustinus, Bergson, Heidegger, etc.) tried to understand observer’s time. However, from the scientific point of view, their attempts may be reckless. From the measurement theoretical point of view, we feel sympathy for J. McTaggart, whose paradox [18] suggests that observer’s time leads science to inconsistency.

5.7. Linguistic turn (Figure 1:③,⑤)

For the question “Which came first, the world or the language?”, two answers (the realistic world view and the linguistic world view) are possible. However, as mentioned in the (I), measurement theory is in the side of “the language came first” (due to Saussure, Wittgenstein, etc.).

Note that two kinds of linguistic turns (i.e., ③ and ⑤) are asserted in **Figure 1**. That is, reprinting the corresponding part of **Figure 1**, we see:



where ③ is the turn from physics to language, and ⑤ is the turn from “brain” to language. Also, note that the linguistic world view (I) is essentially the same as the following Wittgenstein’s famous statement:

- (T₁) *The limits of my language mean the limits of my world.*

In fact, Schrödinger’s cat (or, the theory of relativity) is out of the world described by measurement

theory. And moreover, the assertion (F) is similar to Kant's main assertion ("synthetic a priori judgement") in his famous book "Critique of Pure Reason [20]", that is, the two are similar in the sense of no experimental validation. Therefore, we want to consider the following correspondence:

$$(T_2) \quad \begin{array}{c} [\text{MT}] \\ (\text{Axioms 1 and 2}) \end{array} \leftrightarrow \begin{array}{c} [\text{Critique of Pure Reason}] \\ (\text{synthetic a priori judgement}) \end{array}$$

However, as mentioned in (Q₃), Kant's epistemological approach (i.e., Copernican turn) is rather psychological and not linguistic, in spite that his purpose is philosophy (i.e., the world view) and not psychology (or, brain science).

We (as well as most scientists) consider:

$$(T_3) \quad \text{There is no boundary between "mind" and "matter".}$$

However, this fact should not be confused with our assertion (F), that is,

$$(T_4) \quad \text{Most science should be described by measurement theory (i.e., the language based on dualism (= **Figure 1**)).}$$

If there is the confusion between (T₃) and (T₄), this is due to the misreading of [20].

5.8. There are no facts, only interpretations

Nietzsche's famous saying "There are no facts, only interpretations" is effective in measurement theory. This is shown in this section.

Let testees drink water with various temperature ω °C ($0 \leq \omega \leq 100$). And you ask them "cold" or "hot" alternatively. Gather the data, (for example, $g_c(\omega)$ persons say "cold", $g_h(\omega)$ persons say "hot") and normalize them, that is, get the polygonal lines such that

$$f_c(\omega) = \frac{g_c(\omega)}{\text{the numbers of testees}}$$

$$f_h(\omega) = \frac{g_h(\omega)}{\text{the numbers of testees}}$$

And

$$f_c(\omega) = \begin{cases} 1 & (0 \leq \omega \leq 10) \\ \frac{70-\omega}{60} & (10 \leq \omega \leq 70) \\ 0 & (70 \leq \omega \leq 100) \end{cases},$$

$$f_h(\omega) = 1 - f_c(\omega)$$

Therefore, for example,

$$(U_1) \quad \text{You choose one person from the testees, and you ask him/her "cold" or "hot" alternatively. Then the probability that he/she says ["cold" "hot"] is given by } \begin{bmatrix} f_c(55) = 0.25 \\ f_h(55) = 0.75 \end{bmatrix}$$

This is described in terms of Axiom 1 in what follows. Consider the state space $\mathcal{M}_{+1}^p(\Omega)$ ($\approx \Omega = \text{interval } [0, 100] (\subset \mathbb{R})$) and measured value space $X = \{c, h\}$. Then, we have the (temperature) observable $O_{ch} = (X, 2^X, F_{ch})$ in $C(\Omega)$ such that

$$\begin{aligned} [F_{ch}(\emptyset)](\omega) &= 0, & [F_{ch}(X)](\omega) &= 1 \\ [F_{ch}(\{c\})](\omega) &= f_c(\omega), & [F_{ch}(\{h\})](\omega) &= f_h(\omega) \end{aligned}$$

Thus, we get a measurement $M_{C(\Omega)}(O_{ch}, S_{[\delta_\omega]})$. Therefore, for example, putting $\omega = 55$ °C, we can, by Axiom 1, represent the statement (U₁) as follows.

$$(U_2) \quad \text{the probability that a measured value } x(\in X = \{c, h\}) \text{ obtained by measurement}$$

$$M_{C(\Omega)}(O_{ch}, S_{[\delta_{55}]}) \text{ belongs to set } \begin{bmatrix} \emptyset \\ \{c\} \\ \{h\} \\ \{c, h\} \end{bmatrix} \text{ is}$$

$$\text{given by } \begin{bmatrix} [F_{ch}(\emptyset)](55) = 0 \\ [F_{ch}(\{c\})](55) = 0.25 \\ [F_{ch}(\{h\})](55) = 0.75 \\ [F_{ch}(\{c, h\})](55) = 1 \end{bmatrix}$$

For example, assume that $\omega = 5 \in \Omega (= [0, 100])$. Then, we see, by Axiom 1, that

$$(U_3) \quad \text{the probability that a measured value } x(\in X = \{c, h\}) \text{ obtained by the measurement}$$

$$M_{C(\Omega)}(O_{ch}, S_{[\delta_5]}) \text{ belongs to set } \begin{bmatrix} \emptyset \\ \{c\} \\ \{h\} \\ \{c, h\} \end{bmatrix} \text{ is}$$

$$\text{given by } \begin{bmatrix} [F_{ch}(\emptyset)](5) = 0 \\ [F_{ch}(\{c\})](5) = 1 \\ [F_{ch}(\{h\})](5) = 0 \\ [F_{ch}(\{c, h\})](5) = 1 \end{bmatrix}$$

That is,

$$(U_4) \quad \text{a measured value } x(\in X = \{c, h\}) \text{ obtained by measurement } M_{C(\Omega)}(O_{ch}, S_{[\delta_5]}) \text{ is surely equal to "c".}$$

Here, we must not consider the following causality:

$$(U_5) \quad \boxed{5 \text{ }^\circ\text{C}} \xrightarrow{(\text{causality})} \boxed{\text{cold}} \quad \begin{matrix} (\text{cause}) & & (\text{result}) \end{matrix}$$

The (U₄) is not related to causality but measurement. That is because Axiom 2 (causality) is not used in (U₄). Recall that

- (U₆) Interpretation (H₃) says that causality belongs to the side of system (and not between observer and system).

Thus, the (U₅) is not proper in the above situation.

However, there is another idea as follows. That is, consider the dual causal operator $\Phi^* : \mathcal{M}_{+1}^p([0, 100]) \rightarrow \mathcal{M}_{+1}^m(\{c, h\})$ such that

$$[\Phi^* \delta_\omega](D) = f_c(\omega) \cdot \delta_c(D) + f_h(\omega) \cdot \delta_h(D) \\ (\forall \omega \in [0, 100], \forall D \subseteq \{c, h\})$$

Then, the above (U₅) can be regarded as the causality. Considering the exact observable $O^{(exa)}$ ($= (\{c, h\}, 2^{\{c, h\}}, F_{exa})$) in $C(\{c, h\})$ (where $[F_{exa}(\Xi)](\omega) = 1$ ($\omega \in \Xi$), $= 0$ ($\omega \notin \Xi$)), we see that $O_{ch} = \Phi O^{(exa)}$, and thus, we get the measurement $M_{C(\Omega)}(\Phi O^{(exa)}, S_{[\delta_5]})$. In this case, the above (U₅) is regarded as the causality. In this sense, the (U₅) is proper.

In the above argument, readers may be reminded of Nietzsche's famous saying:

- (U₇) There are no facts, only interpretations.

Also, note that “the measurement of a measurement” is meaningless in measurement theory.

5.9. Parmenides and Zeno

About 2500 years ago, Parmenides said that

- (V₁) *There are no “plurality”, but only “one”. And therefore, there is no movement.*

We want to regard this (V₁) as the origin of Interpretation (H₂) (i.e., “Only one measurement is permitted. And therefore, a state never moves.”).

Zeno, the student of Parmenides, proposed several paradoxes concerning movement. The following “Achilles and the tortoise” is most famous.

[Achilles and the tortoise] *In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead.*

Beginners of philosophy may have a question:

- (V₂) Why have philosophers investigated such an easy problems during 2500 years?

However, it should be noted that “Achilles and the tortoise” is not an elementary mathematical problem concerning geometric series. Since Parmenides and Zeno were philosophers, it is natural to consider that Zeno's paradoxes should be regarded as the problem concerning world view. That is, we believe that Zeno's question is as follows:

- (W) In what kind of world view (in **Figure 1**) should Zeno's paradoxes be understood? And further, if the proper world view is not in **Figure 1**, propose the new world view in which Zeno's paradoxes can be discussed!

It is clear that Zeno's paradoxes are not in physics, and thus, Newtonian mechanics, quantum mechanics and the theory of relativity and so on are not proper for the answer to the problem (W). We assert that classical measurement theory is the proper world view, in which Zeno's paradoxes are described. The classical measurement theoretical description of Zeno's paradoxes is easy, in fact, it was simply shown in [11].

Readers may be interested in the unnatural situation such that “Achilles” and “tortoise” are quantum particles (i.e., Zeno's paradoxes in quantum mechanics). However we have no clear answer to this problem though our paper [21] may be helpful. Also, for the more unnatural situation (i.e., Zeno's paradoxes in the theory of relativity), see [22].

It is interesting and strange to see that we already have the world description methods (i.e., Newtonian mechanics, quantum mechanics and the theory of relativity) for the unnatural situations, but, we have no world description method for the natural situation if we do not know measurement theory. Thus, if we believe in **Figure 1**, we are able to be convinced that Zeno's paradoxes are solved.

5.10. Syllogism

As an example of syllogism, the following example (due to Aristotle) is famous.

- (X₁) *Since Socrates is a man and all men are mortal, it follows that Socrates is mortal.*

However, it should be noted that there is a great gap between the (X₁) and the following mathematical syllogism:

- (X₂) $A \Rightarrow B, B \Rightarrow C$, then, it follows that $A \Rightarrow C$.

That is because the (X₂) is merely mathematical rule and not the world view, and therefore, it is not

guaranteed that the rule (X₂) is applicable to the world (X₁). In fact, as mentioned in ref. [5], the (X₁) does not hold in quantum cases.

Now, we are in the same situation such as (W). That is, we have a similar question (i.e., (W) \approx (W')):

- (W') In what kind of world view (in **Figure 1**) should the phenomenon (X₁) be described? And further, if the proper world view is not in **Figure 1**, propose the new world view!

We assert that classical measurement theory is the proper world view. In fact, in [7], the phenomenon (X₁) is described as a theorem in classical measurement theory.

6. Heisenberg's Uncertainty Principle, EPR-Paradox and Syllogism

It is a matter of course that any argument concerning EPR-paradox is not clear unless the interpretation is declared before the argument. In this sense, we believe that A. Einstein wanted to say in [3]:

- (Y) By a hint of EPR-paradox, propose a new quantum interpretation!

Now we have the linguistic interpretation of quantum mechanics, and thus, we can expect to clarify EPR-paradox.

As shown and emphasized in [5], quantum syllogism does not generally hold. We believe that this fact was, for the first time, discovered in EPR-paradox [3]. The reason that we think so is as follows.

Consider the two-particles system composed of particles P_1 and P_2 , which is formulated in a Hilbert space $L^2(\mathbb{R}^2)$. Let $\rho_s(\in \mathfrak{S}^p(B_c(L^2(\mathbb{R}^2))))$ be the EPR-state in EPR-paradox (or, the singlet state in Bohm's situation). Here, consider as follows:

- (Z₁) Assume that (x_1, p_2) and p'_2 are obtained by the simultaneous measurement of [the position of P_1 , the momentum of P_2] and [the momentum of P_2]. Since it is clear that $p_2 = p'_2$, thus, we see that

$$\begin{array}{c} (x_1, p_2) \\ \text{[the position of } P_1, \text{ the momentum of } P_2 \text{]}) \\ \\ \implies \\ p_2 \\ \text{[the momentum of } P_2 \text{]} \end{array}$$

Here, for the definition of " \implies ", see ref. [5], or more precisely, [7].

- (Z₂) Assume that p_1 and p_2 are obtained by the simultaneous measurement of [the momentum of P_1] and [the momentum of P_2]. Since the state $\rho_s(\in \mathfrak{S}^p(B_c(L^2(\mathbb{R}^2))))$ is the EPR-state, we see that $p_1 = -p_2$, that is, we see that

$$\begin{array}{c} p_2 \\ \text{[the momentum of } P_2 \text{]} \end{array} \implies \begin{array}{c} -p_2 \\ \text{[the momentum of } P_1 \text{]} \end{array}$$

- (Z₃) Therefore, if quantum syllogism holds, (Z₁) and (Z₂) imply that

$$\begin{array}{c} -p_2 \\ \text{[the momentum of } P_1 \text{]} \end{array}$$

that is, the momentum of P_1 is equal to $-p_2$.

Since the above (Z₁)-(Z₃) is not the approximately simultaneous measurement (*cf.* the definition (N)), it is not related to Heisenberg's uncertainty principle (8). Thus, the conclusion (Z₃) is not contradictory to Heisenberg's uncertainty principle (8). This was asserted in the remark 3 of ref. [1]. However, now we can say that the conclusion (Z₃) is not true. That is because the interpretation (H₂) (i.e., only one measurement is permitted) says, as seen in [5], that quantum syllogism does not hold by the non-commutativity of the above three observables, i.e.,

$$\left\{ \begin{array}{l} \text{[the position of } P_1, \text{ the momentum of } P_2 \text{]} \\ \text{[the momentum of } P_2 \text{]} \\ \text{[the momentum of } P_1 \text{]} \end{array} \right\}$$

Thus we see that EPR-paradox is closely related to the fact that quantum syllogism does not hold in general. This should be compared with Bell's inequality [4, 23], which is believed to be closely connected to "non-locality".

7. Conclusions

As mentioned in [4–11], measurement theory (including several conventional system theories, e.g., statistics, dynamical system theory, quantum system theory, etc.) is one of the most useful theories in science. Following the well-known proverb: "A sound mind in a sound body", we consider: "A good philosophy in a very useful theory." For example, "A good realistic philosophy in the theory of relativity" is clearly sure. Therefore, we believe that a good philosophy has to be hidden behind measurement theory. This belief makes us write this paper.

That is, we want to assert “A good linguistic philosophy in quantum mechanics”.

Dr. Hawking said in his best seller book [24]: *Philosophers reduced the scope of their inquiries so much that Wittgenstein the most famous philosopher this century, said “The sole remaining task for philosophy is the analysis of language.” What a come-down from the great tradition of philosophy from Aristotle to Kant!* We think that this is not only his opinion but also most scientists’ opinion. And moreover, we mostly agree with him. However, we believe that it is worth reconsidering the series ① (i.e., the linguistic world view) in **Figure 1**. In spite of Lord Kelvin’s saying: “*Mathematics is the only good metaphysics*”, we assert that measurement theory is also good scientific metaphysics. In order to answer the problem (B₂) (or, Zeno’s paradoxes and so on), we believe that measurement theory is indispensable for science.

In this paper, we see, in the linguistic interpretation of quantum mechanics, that EPR-paradox is closely related to the fact that quantum syllogism does not hold in general. If we can believe that Bell’s inequality is related to “non-locality”, we can clearly understand the difference between EPR-paradox and Bell’s inequality.

We hope that our proposal will be examined from various view points.

8. References

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